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LETTER TO THE EDITOR

A description of the superalgebra $osp(2n+1/2m)$ via Green generators

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Abstract. An alternative description of the Lie superalgebra $osp(2n+1/2m)$ in terms of generators and relations is given. The generators, called Green generators, are the root vectors of $osp(2n+1/2m)$, corresponding to the orthogonal roots.

The root system of the orthosymplectic Lie superalgebra $osp(2n+1/2m)$ reads [1]

$$\Delta = \{\xi\varepsilon_i + \eta\varepsilon_j; \xi\varepsilon_i; 2\xi\varepsilon_k | i \neq j = 1, \dots, m+n \equiv N; k = 1, \dots, m; \xi, \eta = \pm\}. \tag{1}$$

The roots $\varepsilon_1, \dots, \varepsilon_N$ are orthogonal with respect to the Killing form on $osp(2n+1/2m)$. In the present letter we describe the universal enveloping algebra $U[osp(2n+1/2m)]$ of $osp(2n+1/2m)$ in terms of the root vectors $a_i^{\pm} \equiv e_{\mp\varepsilon_i}$, corresponding to $\mp\varepsilon_i$, $i = 1, \dots, N$. All $a_i^{\pm}$ are homogeneous elements with $\boldsymbol{Z}_2 \equiv \{\bar{0}, \bar{1}\}$-grading

$$\deg(a_i^{\pm}) \equiv \langle i \rangle = \begin{cases} \bar{1} & \text{for } i \leqslant m \\ \bar{0} & \text{for } i > m. \end{cases} \tag{2}$$

Our main result is contained in the following theorem.

Theorem. $U[osp(2n+1/2m)]$ is an associative (complex) superalgebra with 1, generators

$$a_1^{\pm}, a_2^{\pm}, \dots, a_{m-1}^{\pm}, a_m^{\pm}, a_{m+1}^{\pm}, \dots, a_{m+n}^{\pm} \equiv a_N^{\pm} \tag{3}$$

relations

$$[\![[\![a_i^{\eta}, a_j^{-\eta}]\!], a_k^{\eta}]\!] = 2\eta^{\langle k \rangle}\delta_{jk}a_i^{\eta} \qquad \forall |i-j| \leqslant 1 \qquad \eta = \pm \tag{4a}$$

$$\{[a_{m-1}^{-\eta}, a_{m+1}^{\eta}], [a_m^{-\eta}, a_{m+2}^{\eta}]\} = 0 \tag{4b}$$

$$[[a_{N-1}^{\eta}, a_N^{\eta}], a_N^{\eta}] = 0 \qquad \eta = \pm \tag{4c}$$

and $\boldsymbol{Z}_2$-grading induced from (2).

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Throughout $[x, y] = xy - yx$, $\{x, y\} = xy + yx$, $[\![x, y]\!] = xy - (-1)^{\deg(x)\deg(y)} yx$. This theorem extends the results of [2], where we have shown that $osp(2n + 1/2m)$ is generated by operators (3), which satisfy the relations

$$[\![[\![a_i^\xi, a_j^\eta]\!], a_k^\epsilon]\!] = 2\epsilon^{\langle k\rangle}\delta_{jk}\delta_{\epsilon,-\eta}a_i^\xi - 2\epsilon^{\langle k\rangle}(-1)^{\langle j\rangle\langle k\rangle}\delta_{ik}\delta_{\epsilon,-\xi}a_j^\eta \qquad i, j, k = 1, \ldots, N \qquad \xi, \eta, \varepsilon = \pm. \tag{5}$$

Equations (5) are among the supercommutation relations of all Cartan–Weyl generators

$$a_i^\xi, [\![a_j^\eta, a_k^\varepsilon]\!] \qquad i, j, k = 1, \ldots, N \qquad \xi, \eta, \varepsilon = \pm. \tag{6}$$

The rest of the supercommutation relations follow from (5) and the (graded) Jacoby identity ($i, j, k, l = 1, \ldots, N$; $\xi, \eta, \varepsilon, \varphi = \pm$):

$$[\![[\![a_i^\xi, a_j^\eta]\!], [\![a_k^\epsilon, a_l^\varphi]\!]]\!] = 2\epsilon^{\langle k\rangle}\delta_{jk}\delta_{\epsilon,-\eta}[\![a_i^\xi, a_l^\varphi]\!] - 2\epsilon^{\langle k\rangle}(-1)^{\langle j\rangle\langle k\rangle}\delta_{ik}\delta_{\epsilon,-\xi}[\![a_j^\eta, a_l^\varphi]\!] - 2\varphi^{\langle l\rangle}(-1)^{\langle j\rangle\langle k\rangle}\delta_{jl}\delta_{\varphi,-\eta}[\![a_i^\xi, a_k^\epsilon]\!] + 2\varphi^{\langle l\rangle}(-1)^{\langle i\rangle\langle j\rangle+\langle i\rangle\langle k\rangle}\delta_{il}\delta_{\varphi,-\xi}[\![a_j^\eta, a_k^\epsilon]\!]. \tag{7}$$

Relations (4) are a small part of (5) and (7). According to the above theorem, however, all equations (5) and (7) follow from (4). Therefore relations (4) contain all information about $osp(2n + 1/2m)$. This is essentially our new result.

The operators (3) with relations (5) are closely related to the generalized quantum statistics, introduced by Green [3]: $a_1^\pm, \ldots, a_m^\pm$ are para-Bose operators (parabosons), whereas $a_{m+1}^\pm, \ldots, a_{m+n}^\pm$ are para-Fermi operators (parafermions). Note that the parabosons do not commute with the parafermions (in the Fock representaton the bosons anticommute with the fermions [2]).

We refer to any set of generators (3) with relations (4) as *Green generators*. $G(n/m)$ denotes the *Green superalgebra*, namely the (free) associative algebra with unity 1, generators (3), relations (4) and grading (2). The theorem asserts that $U[osp(2n+1/2m)] = G(n/m)$.

One could prove the theorem by deriving (5) directly from (4). Here we give another proof. To this end we recall the Chevalley definition of $U[osp(2n+1/2m)]$ and write down explicit relations between the Green and the Chevalley generators. Let (α_{ij}), $i, j = 1, \ldots, N$ be an $N \times N$ symmetric Cartan matrix chosen as [4]

$$(a_{ij}) = (-1)^{\langle j\rangle}\delta_{i+1,j} + (-1)^{\langle i\rangle}\delta_{i,j+1} - [(-1)^{\langle j+1\rangle} + (-1)^{\langle j\rangle}]\delta_{ij} + \delta_{i,m+n}\delta_{j,m+n}. \tag{8}$$

Then $U[osp(2n + 1/2m)]$ is defined as an associative superalgebra with 1 in terms of a number of generators subject to relations. The generators are the Chevalley generators h_i, e_i, f_i ($i = 1, \ldots, N$); the relations are the Cartan–Kac relations

$$[h_i, h_j] = 0 \tag{9a}$$

$$[h_i, e_j] = a_{ij}e_j \tag{9b}$$

$$[h_i, f_j] = -a_{ij}f_j \tag{9c}$$

$$[\![e_i, f_j]\!] = \delta_{ij}h_i \tag{9d}$$

the e-Serre relations

$$(e1)\ [e_i, e_j] = 0 \quad \text{for } |i-j| > 1 \qquad (e2)\ [\![e_i, [e_i, e_{i\pm1}]]\!] = 0 \quad i \neq N$$
$$(e3)\ \{[e_{m-1}, e_m], [e_m, e_{m+1}]\} = 0 \qquad (e4)\ [e_N, [e_N, [e_N, e_{N-1}]]] = 0 \tag{10}$$

and the f-Serre relations obtained from (10) by replacing e_i with f_i everywhere. The grading on $U[osp(2n + 1/2m)]$ is induced from $\deg(e_m) = \deg(f_m) = \bar{1}$, $\deg(e_i) = \deg(f_i) = \bar{0}$ for $i \neq m$.

Let us only note the connection to the root system (1). The $3N$ Chevalley elements are the simple roots $h_j = \varepsilon_j - \varepsilon_{j+1}$, $j = 1, \ldots, N-1$, $h_N = \varepsilon_N$, the simple root vectors

$e_i \equiv e_{h_i}$ and the negative root vectors $f_i \equiv e_{-h_i}$, $i = 1, \ldots, N$. Then $(h_i, h_j) = \alpha_{ij}$ or equivalently $(\varepsilon_i, \varepsilon_j) = -(-1)^{\langle i \rangle}\delta_{ij}$ [4].

Introduce the following elements in $U[osp(2n+1/2m)]$ $(i \neq N)$:

$$a_i^- = (-1)^{(m-i)\langle i \rangle}\sqrt{2}[e_i, [e_{i+1}, [\ldots, [e_{N-2}, [e_{N-1}, e_N]]\ldots]]] \qquad a_N^- = \sqrt{2}e_N \tag{11a}$$

$$a_i^+ = -\sqrt{2}[f_i, [f_{i+1}, [\ldots, [f_{N-2}, [f_{N-1}, f_N]]\ldots]]] \qquad a_N^+ = -\sqrt{2}f_N \tag{11b}$$

which will be refered to as *creation ($\xi = +$) and annihilation ($\xi = -$) operators* (CAOs).

The proof of the theorem will be a consequence of a few propositions. The aim is to show that the relations among the Chevalley generators hold if and only if the CAOs (11) are Green generators.

Proposition 1. For any $i \neq N$

$$[\![e_i, a_j^+]\!] = -(-1)^{\langle j+1 \rangle}\delta_{ij}a_{j+1}^+ \tag{12a}$$

$$[\![f_i, a_j^-]\!] = -(-1)^{\langle i \rangle + \langle i+1 \rangle}\delta_{ij}a_{j+1}^- \tag{12b}$$

$$[\![e_i, a_j^-]\!] = -(-1)^{\langle j \rangle}\delta_{i+1,j}a_{j-1}^- \tag{12c}$$

$$[\![f_i, a_j^+]\!] = \delta_{i+1,j}a_{j-1}^+. \tag{12d}$$

Proof. We skip the proof of (12a) and (12b). It is simple and follows only from (9), ($e1$) and ($f1$). Here are some key points in the proof of (12d).

(i) The case $i < j-1$ follows only from ($f1$); the case $i = j-1$ reduces to the definition of a_i^+; the case $i = j$ is derived relatively easily from ($f1$) and ($f2$).

(ii) The case $i = m > j$. We know from above that $[f_{m-1}, a_{m+2}^+] = [f_m, a_{m+2}^+] = 0$. Therefore

$$a_{m-1}^+ = [f_{m-1}, [f_m[f_{m+1}, a_{m+2}^+]]] = [[f_{m-1}, [f_m, f_{m+1}]], a_{m+2}^+].$$

Hence

$$[\![f_m, a_{m-1}^+]\!] = [\![f_m, [[f_{m-1}, [f_m, f_{m+1}]], a_{m+2}^+]]\!] = [\![z, a_{m+2}^+]\!]$$

where

$$\begin{aligned} z &= [\![f_m, [\![f_{m-1}, [\![f_m, f_{m+1}]\!]]\!]]\!] \\ &= [\![[\![f_m, f_{m-1}]\!], [\![f_m, f_{m+1}]\!]]\!] + [\![f_{m-1}, [\![f_m, [\![f_m, f_{m+1}]\!]]\!]]\!] \\ &= \{[f_{m-1}, f_m], [f_m, f_{m+1}]\} + [f_{m-1}, \{f_m, [f_m, f_{m+1}]\}] \\ &= 0 \end{aligned}$$

according to ($f2$) and ($f3$). This is the only place one uses the Serre relation ($f3$), which was recently established [5]. Thus $[\![f_m, a_{m-1}^+]\!] = 0$. Then by induction on j one obtains that $[\![f_m, a_j^+]\!] = 0$ for any $m > j$.

(iii) The case $i \neq m$, $i > j$. Let $1 < i < N-1$. From above (since $\deg(f_i) = \bar{0}$) $[f_{i-1}, a_{i+2}^+] = [f_i, a_{i+2}^+] = 0$. Therefore

$$[\![f_i, a_{i-1}^+]\!] = [f_i, a_{i-1}^+] = [f_i, [f_{i-1}, [f_i, [f_{i+1}, a_{i+2}^+]]]] = [y, a_{i+2}^+]$$

where

$$y = [f_i, [f_{i-1}, [f_i, f_{i+1}]]] = -[[[f_{i+1}, f_i], f_{i-1}], f_i].$$

Now we use the following identity: if $[a, c] = 0$, then

$$[[[c, b], a], b] = \tfrac{1}{2}[[[c, b], b], a] + \tfrac{1}{2}[[[a, b], b], c].$$

It yields

$$y = -[[[f_{i+1}, f_i], f_{i-1}], f_i] = -\tfrac{1}{2}[[[f_{i+1}, f_i], f_i], f_{i-1}] - \tfrac{1}{2}[[[f_{i-1}, f_i], f_i], f_{i+1}] = 0$$

since from $(f2)$ $[[f_{i\pm1}, f_i], f_i] = 0$. If $i = N-1$ the proof is even simpler. Thus, $[\![f_i, a_{i-1}^+]\!] = 0$ for any $i \neq m$. Let $m \neq i > j+1$. Assume $[f_i, a_j^+] = 0$. Then

$$[f_i, a_{j-1}^+] = [f_i, [f_{j-1}, a_j^+]] = [[f_i, f_{j-1}], a_j^+] + [f_{j-1}, [f_i, a_j^+]] = 0.$$

Hence $[f_i, a_j^+] = 0$ for each $m \neq i > j$ $(i \neq N)$.

From (i)–(iii) follows (12d). The derivation of (12c) is similar. This completes the proof. □

Proposition 2. $U[osp(2n+1/2m)]$ is generated by the CAOs (11). More precisely,

$$h_N = -\tfrac{1}{2}[\![a_N^-, a_N^+]\!] \qquad e_N = \frac{1}{\sqrt{2}}a_N^- \qquad f_N = -\frac{1}{\sqrt{2}}a_N^+ \tag{13a}$$

$$\begin{aligned} h_i &= \tfrac{1}{2}[\![a_{i+1}^-, a_{i+1}^+]\!] - \tfrac{1}{2}[\![a_i^-, a_i^+]\!] \qquad e_i = \tfrac{1}{2}[\![a_i^-, a_{i+1}^+]\!] \\ f_i &= \tfrac{1}{2}[\![a_i^+, a_{i+1}^-]\!] \qquad i \neq N. \end{aligned} \tag{13b}$$

Proof. Equations (13a) are evident. From (9) and (11) $[\![a_{N-1}^-, a_N^+]\!] = -2[[e_{N-1}, e_N], f_N] = -2[e_{N-1}, [e_N, f_N]] = 2[h_N, e_{N-1}] = 2e_{N-1}$. Thus expressions (13$b$) for e_{N-1} and similarly for f_{N-1} hold. Then

$$\begin{aligned} \tfrac{1}{2}[\![a_N^-, a_N^+]\!] - \tfrac{1}{2}[\![a_{N-1}^-, a_{N-1}^+]\!] &= -h_N - \frac{1}{\sqrt{2}}[\![[\![e_{N-1}, e_N]\!], a_{N-1}^+]\!] \\ &= -h_N - \frac{1}{\sqrt{2}}[\![[\![e_{N-1}, a_{N-1}^+]\!], e_N]\!] - \frac{1}{\sqrt{2}}[\![e_{N-1}, [\![e_N, a_{N-1}^+]\!]]\!] \\ &= h_{N-1} \end{aligned}$$

and (13b) holds for $i = N-1$. The rest is proved by induction. Assume (13b) holds for $k = i+1, i+2, \ldots, N-1$. Then

$$\begin{aligned} \tfrac{1}{2}[\![a_i^-, a_{i+1}^+]\!] &= \tfrac{1}{2}(-1)^{\langle i+1\rangle}[\![[\![e_i, a_{i+1}^-]\!], a_{i+1}^+]\!] \\ &= \tfrac{1}{2}(-1)^{\langle i+1\rangle}[\![e_i, [\![a_{i+1}^-, a_{i+1}^+]\!]]\!] \\ &= -(-1)^{\langle i+1\rangle}[\![e_i, h_{i+1} + h_{i+2} + \cdots + h_N]\!] \\ &= e_i \end{aligned}$$

similarly one verifies (13b) for f_i. Finally

$$\begin{aligned} \tfrac{1}{2}[\![a_{i+1}^-, a_{i+1}^+]\!] - \tfrac{1}{2}[\![a_i^-, a_i^+]\!] &= \tfrac{1}{2}[\![a_{i+1}^-, a_{i+1}^+]\!] - \tfrac{1}{2}(-1)^{\langle i+1\rangle}[\![[\![e_i, a_{i+1}^-]\!], a_i^+]\!] \\ &= \tfrac{1}{2}[\![a_{i+1}^-, a_{i+1}^+]\!] - \tfrac{1}{2}[\![[\![e_i, a_i^+]\!], a_{i+1}^-]\!] - \tfrac{1}{2}(-1)^{\langle i+1\rangle}[\![e_i, [\![a_{i+1}^-, a_i^+]\!]]\!] \\ &= [\![e_i, f_i]\!] \\ &= h_i. \end{aligned}$$

Hence equations (13b) hold for any $i \neq N$. □

Proposition 3. The CAOs (11) are Green generators, i.e. they satisfy equations (4).

Proof. Replacing e_i and f_i in (12) with the corresponding expressions from (13b), one derives equations (4a) for all $|i-j| = 1$ and $\eta = \pm$. Equation (4a), corresponding to $i = j$, follows from $[\![a_i^-, a_i^+]\!] = -2(h_i + h_{i+1} + \cdots + h_N)$, $i = 1, 2, \ldots, N$, the Cartan–Kac relations (9) and the Cartan matrix (5). Equations (4b, c) follow immediately from the Serre relations $(e3)$, $(f3)$, $(e4)$, $(f4)$ (see (10)) and the definition (11) of the CAOs. □

So far we have established that the operators (11) can be considered as new generating elements of $U[osp(2n+1/2m)]$ (proposition 2), which satisfy the Green relations (4) (proposition 3). Therefore $U[osp(2n+1/2m)]$ is either equal to the Green algebra $G(n/m)$ or is a factor algebra of it. In order to show that $U[osp(2n+1/2m)] = G(n/m)$ we need to prove that any relation in $U[osp(2n+1/2m)]$ is a consequence of the Green relations. To this end it suffices to show that equations (9) and (10), expressed in terms of the CAOs according to (13), can be derived from (4).

Proposition 4. Let $a_i^\pm$, $i = 1, \ldots, N$ be Green generators, i.e. they obey relations (4). Then the operators h_i, e_i, f_i $(i = 1, \ldots, N)$, defined with (13), satisfy equations (9) and (10).

Proof. The proof resolves into several cases. We mention some of them. Equation (9a) is trivial. Consider for instance (9b) for $i, j \neq N$. From the graded Jacoby identity one has

$$\begin{aligned}[h_i, e_j] &= \tfrac{1}{4}[[\![a_{i+1}^-, a_{i+1}^+]\!], [\![a_j^-, a_{j+1}^+]\!]] - \tfrac{1}{4}[[\![a_i^-, a_i^+]\!], [\![a_j^-, a_{j+1}^+]\!]] \\ &= \tfrac{1}{4}[\![[\![[\![a_{i+1}^-, a_{i+1}^+]\!], a_j^-]\!], a_{j+1}^+]\!] + \tfrac{1}{4}[\![a_j^-, [\![[\![a_{i+1}^-, a_{i+1}^+]\!], a_{j+1}^+]\!]]\!] \\ &\quad -\tfrac{1}{4}[\![[\![[\![a_i^-, a_i^+]\!], a_j^-]\!], a_{j+1}^+]\!] - \tfrac{1}{4}[\![a_j^-, [\![[\![a_i^-, a_i^+]\!], a_{j+1}^+]\!]]\!].\end{aligned}$$

Applying (4) here one ends up with (9b). The other cases of (9) are similar or simpler. Special care should be taken for the grading of the operators that appear. To check (10) one needs $(i, j \neq N)$

$$\begin{aligned}[\![e_i, e_j]\!] &= \tfrac{1}{4}[\![[\![a_i^-, a_{i+1}^+]\!], [\![a_j^-, a_{j+1}^+]\!]]\!] \\ &= \tfrac{1}{4}(-1)^{[\langle i\rangle+\langle i+1\rangle+]\langle j\rangle}[\![a_j^-, [\![[\![a_i^-, a_{i+1}^+]\!], a_{j+1}^+]\!]]\!] + \tfrac{1}{4}[\![[\![[\![a_i^-, a_{i+1}^+]\!], a_j^-]\!], a_{j+1}^+]\!]\end{aligned}$$

which, taking into account that $(-1)^{[\langle i\rangle+\langle i+1\rangle]\langle i-1\rangle+\langle i\rangle\langle i+1\rangle} = (-1)^{\langle i\rangle}$, finally yields

$$\begin{aligned}[\![e_i, e_j]\!] &= \tfrac{1}{2}(-1)^{\langle i+1\rangle}\delta_{i+1,j}[\![a_i^-, a_{i+2}^+]\!] - \tfrac{1}{2}(-1)^{\langle i\rangle}\delta_{i,j+1}[\![a_{i-1}^-, a_{i+1}^+]\!] \\ &\qquad i, j = 1, \ldots, N-1.\end{aligned} \tag{14}$$

From (14) one derives the Serre relations ($e1$), ($f1$), ($e2$), ($f2$). Similarly ($e3$), ($f3$), ($e4$), ($f4$) follow from (4b). For instance $\{[e_{m-1}, e_m], [e_m, e_{m+1}]\} = -\frac{1}{4}\{[a_{m-1}^-, a_{m+1}^+], [a_m^-, a_{m+2}^+]\} = 0$ and $[e_N, [e_N, [e_N, e_{N-1}]]] = -[[a_{N-1}^-, a_N^-], a_N^-] = 0$. Hence $U[osp(2n+1/2m)] = G(n/m)$, which completes the proof of the proposition and hence of the theorem. □

We have shown that apart from the Chevalley definition the associative superalgebra $U[osp(2n+1/2m)]$ and hence the Lie superalgebra $osp(2n+1/2m)$ allows an alternative description in terms of generators (3) subject to the Green relations (4). This, in particular, means that the Green generators satisfy equations (5), i.e. $a_1^\pm, a_2^\pm, \ldots, a_m^\pm$ are para-Bose operators and $a_{m+1}^\pm, \ldots, a_{m+n}^\pm$ are para-Fermi operators.

Our interest in the present work originates from the study of the Wigner quantum oscillators [6]. The defining relations for an N-dimensional such oscillator are $\sum_{i=1}^N [\{a_i^+, a_i^-\}, a_k^\pm] = \pm 2a_k^\pm$. The operators (5) satisfy these equations for any $m = 1, \ldots, N$ and therefore provide examples of Wigner oscillators. The $osp(3/2)$ oscillator was studied in [7]. Already in this quite simple case the verification of all triple relations (5) is a non-trivial task and it is going to be much more difficult for the general $osp(2n+1/2m)$ oscillator. The theorem now asserts that it is sufficient to check (or, which is more important, to solve) the smaller set of equations (4), which is a considerable simplification. The same arguments also hold in the more general context of the representations of $osp(2n+1/2m)$, which are at present only classified [1] (explicit expressions exist only for $osp(1/2)$ [10], $osp(2/2)$ [11] and $osp(3/2)$ [9]). We hope that relations (4), combined with the generalization of the Green anzatz technique [12], provide a good background for constructing new representations. The same holds for any of the subalgebras $osp(2n/2m)$, $gl(n/m)$, $so(2n+1)$, $so(2n)$, $gl(n)$ and $gl(m)$ since the latter can be expressed in a rather natural way via Green generators (see (2.5)–(2.9) in [9]). These are the pre-oscillator realizations, which in the Fock representation reduce to the known Schwinger realizations.

Recently, it was shown that the Green description with only parafermions ($m = 0$) or with only parabosons ($n = 0$) can be modified to the quantum algebras $U_q[so(2n+1)]$ [13] and $U_q[osp(1/2m)]$ [14]. This leads to natural Hopf algebra deformations of the para-Fermi and of the para-Bose statistics. We believe it will be possible to generalize the Green description to the quantum algebra $U_q[osp(2n+1/2m)]$. The latter would amount to a simultaneous deformation of the parabosons and the parafermions (in the Fock representation, of the bosons and the fermions) as one single supermultiplet.

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